PERFORMANCE ANALYSIS ON RECURSIVE SINGLE-SIDEBAND AMPLITUDE MODULATION FOR PARAMETRIC LOUDSPEAKERS

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ABSTRACT

A highly directional speech signal can be generated using parametric loudspeaker. The generation of highly directional sound beam is due to the nonlinear interaction of amplitude-modulated ultrasound waves in air. However, severe distortion is also generated during the reproduction of directional speech and several preprocessing techniques based on the Berktay's farfield model have been proposed by researchers to reduce the distortion. In this paper, we carried out a thorough investigation on the analytical performance of the recursive single-sideband amplitude modulation (RSSB-AM) technique, which has been found to perform well for directional speech reproduction. Several important characteristics of the performance of the RSSB-AM are observed and optimal parameters of the RSSB-AM are also presented.

Keywords—Directional Sound, Distortion Reduction, Nonlinear System, System Approximation

1. INTRODUCTION

Highly directional sound beams are widely applied in the areas of medical ultrasound, acoustic microscopy, non-destructive testing, and underwater acoustics [1]. Unlike conventional loudspeaker that requires a large speaker array to deliver high directional sound wave, parametric loudspeaker [2-8] with a small aperture has a unique feature in delivering high-directivity low-frequency speech signal to a desired location by utilizing the nonlinear acoustic behavior in air. Berktay [9] has given an accurate and complete explanation of the parametric loudspeaker and also a farfield solution of $p_2(t) \propto \partial^2 E^2(t)/\partial t^2$, assuming the primary wave has the waveform of $p_1(t) = E(t)\sin \omega_c t$, where $E(t)$ is the modulation envelope function, ω_c is the
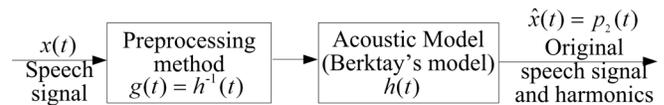
Fig. 1. Block diagram of the operation of a parametric loudspeaker.

angular carrier frequency and $p_2(t)$ is the demodulated signal.

Figure 1 shows a simple block diagram on how speech signal can be preprocessed in a system $g(t)$ before passing to the air modeled by an acoustic model (modeled by the Berktay's model) $h(t)$. To recover the speech signal $x(t)$, the system $g(t)$ must approximate the inverse of the acoustic model, i.e., $g(t) \approx h^{-1}(t)$. There are many approaches [3, 10-15] in estimating the inverse of the acoustic model to achieve an output signal $\hat{x}(t)$ as close as to $x(t)$. However, due to the nonlinear acoustic property in air, nonlinear distortion $d_a(t)$ consisting of harmonics and sum frequencies are generated together with the desired signal $x(t)$. Therefore, we have $\hat{x}(t) = x(t) + d_a(t)$.

A preprocessing technique is used to minimize the distortion $d_a(t)$ and results in a demodulated output signal $\hat{x}(t) = p_2(t)$ which approximates the original desired signal $x(t)$. Hence, the desired signal is heavily distorted if no preprocessing method is applied [15]. To improve the quality of the desired signal, many research works [3, 10-15] have been carried out to determine a preprocessing method that reduces as much distortion as possible. The conventional double-sideband amplitude modulation (DSB-AM) [3] is one of the simplest preprocessing methods. However, this method suffers from high distortion as it does not remove the nonlinear distortion. Several other methods, such as the square root AM (SRAM) [10, 11], the single-sideband AM (SSB-AM) [12] and modified AM (MAM) [13] significantly reduce distortion, but all these methods have their disadvantages, i.e., both the SRAM and MAM need infinite bandwidth and the SSB-AM has high inter-modulation distortion (IMD), which will be discussed in this paper.

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In this paper, we analyze a new class of preprocessing technique, known as the recursive single-sideband AM (RSSB-AM), which was first conceptualized by Croft *et al.* and only the basic ideas of the RSSB-AM were presented in [14, 15]. Since then, there is no report to show how the RSSB-AM is superior to existing methods. In this study, we will present a detailed investigation on the recursive properties of the RSSB-AM, including both the IMD and error analyses. For directional speech reproduction, our observations indicate that the RSSB-AM effectively reduces the distortion found with parametric loudspeakers. As the RSSB-AM is a recursive algorithm, it is important to determine the number of iterations required to achieve the minimum distortion with respect to the modulation index.

The rest of the paper is organized as follows. In Section 2, the RSSB-AM is reviewed. Analytical solution and illustrative explanation for the RSSB-AM with a dual-tone input are also discussed. In Section 3, numerical simulations are carried out to evaluate performance of the RSSB-AM using the IMD and error analyses. Finally, our conclusions are presented in Section 4.

2. RECURSIVE SINGLE-SIDEBAND AMPLITUDE MODULATION

To evaluate the actual performance of the RSSB-AM, we assume that the second-time derivative effect in the Berktay's model can be perfectly compensated by an ultrasonic emitter with a bandwidth of at least 4 kHz. This leads to the demodulated signal $\hat{x}(t)$ proportional to the square of the modulation envelope $E^2(t)$. Thus, we only need to derive the expressions describing the square of the envelope $E^2(t)$ to evaluate the performance of the RSSB-AM.

2.1. RSSB-AM and its Working Principle

The RSSB-AM consists of the SSB modulator and the nonlinear demodulator (NLD), as shown in Figs. 2(a) and 2(b), respectively. The role of the SSB modulator is to modulate the carrier angular frequency ω_c with the input signal, while the NLD calculates the square of the envelope, which models the nonlinear acoustic propagation in air. These two blocks are combined into a distortion model (DM) in Fig. 2(c) and subtracted by the original input $x(t) = x_0(t)$ to obtain the distortion $d_{i-1}(t)$ at ith stage, where $i = 1, 2, \ldots, q$. In the block diagram of the RSSB-AM, as shown in Fig. 2(d), the distortion at every preceding stage, $d_{i-1}(t)$ is combined with the pre-distorted signal $x_{i-1}(t)$ to obtain an input of $x_i(t)$, which serves as the input to the $(i + 1)$th stage. The distortion is progressively reduced by each DM stage; higher reduction is achieved by cascading several DM stages. An analysis of the reduction
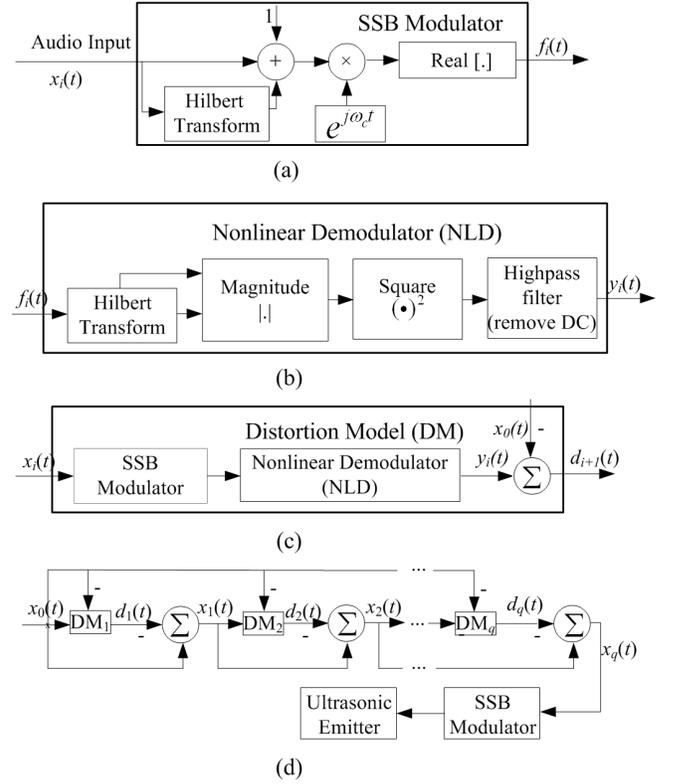

Fig. 2. Block diagram of the RSSB-AM and its components. (a) SSB modulator, (b) NLD, (c) DM and (d) RSSB-AM, where $j = \sqrt{-1}$.

of distortion by each DM stage will be presented in the following sections.

The main computational complexity of the RSSB-AM is given by the Hilbert transform and highpass filter, which is implemented as r_1-tap and r_2-tap FIR filters, respectively. The SSB modulator requires r_1+2 additions and r_1+1 multiplications; NLD requires $r_1+ r_2$ additions and r_1+r_2+1 multiplications, hence, each DM stage requires $2r_1+r_2+3$ additions and $2r_1+r_2+2$ multiplications. For the RSSB-AM having i DM stages, the computational complexity would be $i(2r_1+r_2+4)+r_1+2$ additions and $i(2r_1+r_2+2)+r_1+1$ multiplications. From this discussion, it is clear that the computational complexity of the RSSB-AM is proportional to the number of DM stages and the number of taps in the FIR filters.

The operation of the RSSB-AM is explained as follows. $x_0(t)$ is the speech input signal, and passes to the first DM stage (DM$_1$) to generate the distortion term $d_0(t)$, which is in turn subtracted from the original speech signal to form a pre-distorted signal $x_1(t)$. Then $x_1(t)$ serves as the input to the second stage of the RSSB-AM, and in turn the original signal is subtracted from the output of the NLD to produce the distortion term $d_1(t)$. The pre-distorted signal $x_2(t)$ is generated by subtracting the pre-distorted signal $x_1(t)$ with the distortion term $d_1(t)$. The iterative process continues

Table 1. Relation between input signal, output signal and distortion for ith stage of RSSB-AM ($i = 0, 1, 2, 3$)

RSSB-AM with i^{th} stage	Input signal	Output signal	Distortion
$i=0$ (SSB-AM)	$x_0(t)$	$x_0(t) + m^2\cos(\omega_1 t - \omega_2 t)/4$	$m^2\cos(\omega_1 t - \omega_2 t)/4$
$i=1$	$x_0(t) - m^2\cos(\omega_1 t - \omega_2 t)/4$	$x_0(t) - m^3\cos(\omega_2 t)/8$ $-m^3\cos(\omega_1 t - 2\omega_2 t)/8$	$-m^3\cos(\omega_2 t)/8$ $-m^3\cos(\omega_1 t - 2\omega_2 t)/8$
$i=2$	$x_0(t) - m^2\cos(\omega_1 t - \omega_2 t)/4$ $+m^3[\cos(\omega_2 t) + \cos(\omega_1 t - 2\omega_2 t)]/8$	$x_0(t) + m^4 T[3]/16$ $-m^5 T[2]/32 + m^6 T[1]/64$	$m^4 T[3]/16 - m^5 T[2]/32$ $+m^6 T[1]/64$
$i=3$	$x_0(t) - m^2\cos(\omega_1 t - \omega_2 t)/4$ $+m^3[\cos(\omega_2 t) + \cos(\omega_1 t - 2\omega_2 t)]/8$ $-m^4 T[3]/16 + m^5 T[2]/32$ $-m^6 T[1]/64$	$x_0(t) - m^5 T[6]/32 + m^6 T[6]/64$ $-m^7 T[10]/128 + m^8 T[7]/256$ $-m^9 T[7]/512 + m^{10} T[3]/1024$ $-m^{11} T[2]/2056$	$-m^5 T[6]/32 + m^6 T[6]/64$ $-m^7 T[10]/128 + m^8 T[7]/256$ $-m^9 T[7]/512 + m^{10} T[3]/1024$ $-m^{11} T[2]/2056$

until the distortion at the qth iteration is reduced to a predefined level or the distortion at the ith stage is not less than the $(i-1)$th stage, i.e. $d_{i-1}(t) \geq d_{i-2}(t)$, $i = 2, 3,\ldots,$ q. The distortion at each stage must satisfy the following condition: $d_0(t) \geq \cdots \geq d_{i-1}(t) \geq \cdots \geq d_{q-1}(t)$. In this paper, we examine the amount of distortion reduction with respect to the number of stages and the corresponding computational complexity of the RSSB-AM.

2.2. Single-tone and Dual-tone Analyses

For the single-tone analysis, we define an input $x_0(t) = m\cos(\omega_1 t)$, where m is the modulation index and ω_1 is the speech angular frequency. The output from DM_1 $y_0(t) = m\cos(\omega_1 t)$ results in $d_0(t) = 0$. This simple case implies that there is no need to apply any recursive process when a single tone is applied to the RSSB-AM. In this case, the RSSB-AM is equivalent to the SSB-AM, which does not generate any distortion.

For the dual-tone analysis, we use a signal $x_0(t) = m[\cos(\omega_1 t) + \cos(\omega_2 t)]/2$, where ω_1 and ω_2 are angular frequencies within the speech spectrum. Here, only the analytical expressions for the RSSB-AM are derived for the case of $\omega_1 > \omega_2$. Similar results can be obtained by swapping ω_1 and ω_2 for the case of $\omega_1 < \omega_2$. The RSSB-AM having up to three iterations is shown in Table 1 to illustrate the cancellation of the distortion terms at the expense of introducing more distortion terms with magnitude depending on the value of the modulation index. $T[u]$ denotes a polynomial with u distortion terms, which fall within the bandwidth of the original input signal. The case where $i = 0$ in Table 1 denotes the special case where no iteration is implemented (SSB-AM).

From Table 1, it is observed that the distortion term having the highest magnitude in the ith iteration has a magnitude of $(-1)^i m^{i+2} / 2^{i+2}$. This distortion term is removed by the following DM stage, and replaced by another distortion term of a smaller magnitude as observed in the last column of Table 1. When more iterations of the RSSB-AM are performed, more distortion terms having smaller magnitude appear. In addition, it is observed that all the distortions fall within the bandwidth of the input speech signal. In other words, the RSSB-AM does not require an ultrasonic emitter having large bandwidth and hence only requires half the bandwidth of the DSB-AM. This is one of the main advantages of the RSSB-AM when applied to parametric loudspeakers.

2.3. Illustrative Explanation of RSSB-AM

Figure 3 illustrates the distortion terms produced in the ith iteration of the RSSB-AM. This figure also illustrates the reduction of the distortion in the $(i + 1)$th iteration. The illustrative explanation of the RSSB-AM can help us to better understand the rationale behind the RSSB algorithm. For clarity, only the dual-tone input is used. Uppercase S indicates the frequency component is in the ultrasonic frequency range; and lowercase s is in the speech frequency range.

Starting from non-iterative RSSB-AM ($i = 0$), $x_0(t)$ is directly sent to the SSB modulator, and produces three terms: S_1, S_2, and S_3, where $S_1 = \cos\omega_c t$, $S_2 = m\cos(\omega_c t + \omega_1 t)/2$, and $S_3 = m\cos(\omega_c t + \omega_2 t)/2$. During the propagation in air, nonlinear interaction occurs between any two of these terms, which is modeled by the NLD. The desired signals are reproduced from the interaction between (S_1 and S_2) and (S_1 and S_3). The distortion s_a, corresponding to $d_0(t)$ in Fig. 2(d) is generated from the interaction between S_2 and S_3. This interaction is represented as the solid line with hollow arrow in Fig. 3(a). In the first iteration ($i = 1$), s_a is negated and added to $x_0(t)$ to obtain the pre-distorted
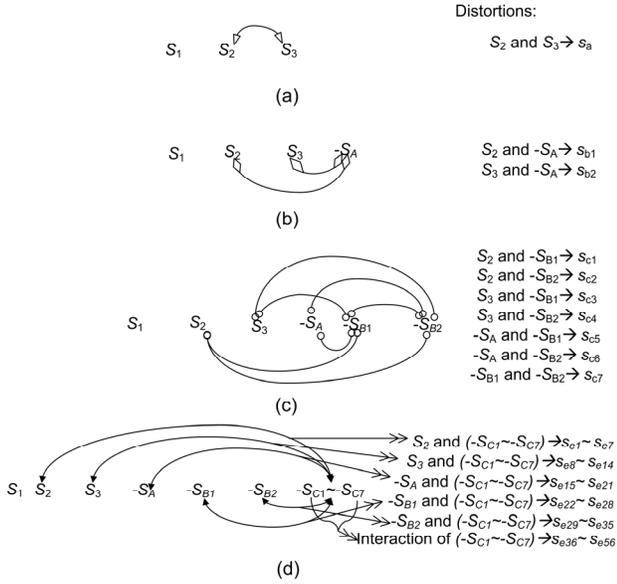
Fig. 3. Illustration explanation for RSSB-AM for dual-tone input with *i*th stage (a) $i = 0$ (SSB-AM), (b) $i = 1$, (c) $i = 2$ and (d) $i = 3$.

signal $x_1(t)$, which is channeled to the SSB modulator and then produced four terms: S_1, S_2, S_3 and $-S_A$, where the appearance of $-S_A$ is due to $-s_a$. In this iteration, the error term s_a is cancelled by the interaction between S_1 and $-S_A$. However, two new (but smaller) distortion terms s_{b1} and s_{b2} are generated. This new distortion terms are generated from the interaction between (S_2 and $-S_A$), and (S_3 and $-S_A$) corresponding to $d_1(t)$ in Fig. 2(d). The formation of these distortion terms s_{b1} and s_{b2} is represented by the solid line with diamond in Fig. 3(b). Similar technique is used to eliminate the two distortion terms s_{b1} and s_{b2} which produces the pre-distorted signal $x_2(t)$ and the modulated signal in the second iteration ($i = 2$), where $x_2(t) = x_1(t) - s_{b1} - s_{b2}$. Although all the distortions terms in the previous iterations are removed, seven new (smaller) distortion terms appear. This formation of the distortion terms is represented as the solid line with circle in Fig. 3(c). These distortion terms are the products due to the nonlinear interaction between two different terms, as listed on the right hand side of Fig. 3(c), which correspond to $d_2(t)$ in Fig. 2(d). As more iterations of the RSSB-AM are performed, more additional terms are required to reduce the distortions.

For $i \geq 4$, the number of distortion terms n_i can be computed as: $3n_{i-1} + \sum_{k=2}^{i-2} n_k n_{i-1} + n_{i-1}(n_{i-1} - 1)/2$. This expression implies that n_i is determined by adding the number of distortion terms in the previous iterations. For example, $n_4 = 56$, $n_5 = 2212$, and $n_6 = 2595782$. For the (i + 1)th iteration, the number of additional terms to be

added to the input is $\sum_{k=1}^{i} n_k$. Due to the huge number of distortion terms, we have only illustrated up to three iterations in Fig. 3. Therefore, the number of distortion terms for $i \leq 3$ (i = 0, 1, 2, 3) is 1, 2, 7 and 56, respectively. However, from Table 1, 1, 2, 6 and 41 distortion terms are observed for the first three iterations. These differences in the second and third iterations are found as the distortion terms having the same frequency in Table 1 are grouped as one single distortion term.

3. SIMULATION RESULTS

To investigate the performance of the RSSB-AM, we first present an IMD analysis using arbitrary dual-tone input of 1 kHz and 1.3 kHz. Different number of iterations are used with the RSSB-AM with the modulation index varies from 0 to 1 at an interval of 0.05. Subsequently, error analysis of the RSSB-AM using one speech signal is carried out for selected modulation indices.

3.1. IMD Analysis

To determine the performance of the RSSB-AM, the IMD index is adopted. In this subsection, angular frequencies $\omega_1 = 2\pi(1.3k)$ and $\omega_2 = 2\pi(1k)$ are used. IMD is computed as $\text{IMD} = \sqrt{V_s^2 - D_{\omega_1}^2 - D_{\omega_2}^2}/V_s$, where D_ω and V_s denote the desired component at angular frequency at ω and amplitude of the speech signal, respectively. The IMD of the DSB-AM [13] is also included for comparison and is calculated as: $\text{IMD} = \sqrt{5}m/\sqrt{64 + 5m^2}$.

From Table 1, it is clear that the amplitude of the desired signals and the distortion terms are all proportional to the modulation index. So the value of modulation index should be carefully chosen to balance the tradeoff between the distortions and the desired signal. The relation between the IMD values and *m* for different number of iteration (up to eight iterations) is shown in Fig. 4. An IMD threshold of 5% is drawn in Fig. 4 which serves as the desired performance benchmark. Initially, the IMD values of the SSB-AM (non-iterative RSSB-AM) are larger than those of the DSB-AM. After one iteration, the RSSB-AM outperforms the DSB-AM. For $m \leq 0.75$, it is also noted that the IMD values of the RSSB-AM decreases as more iterations are used. For $m > 0.75$, the IMD values increase with the number of iterations used. This is due to the following reasons. First, fewer distortion terms are produced in the early iterations of RSSB-AM. Second, the magnitude of the distortion terms is proportional to the modulation index. Hence, this yields serious overmodulation for large
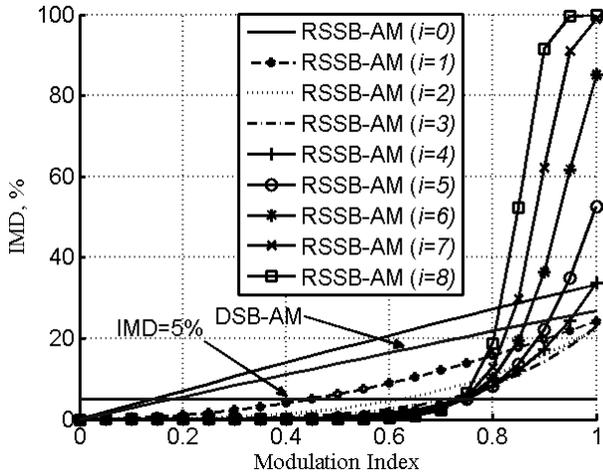
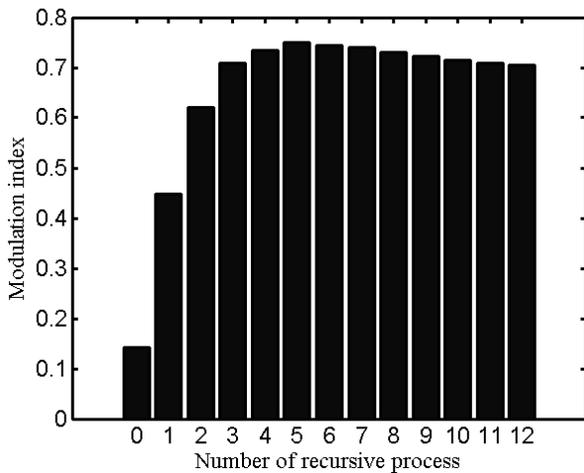**Fig. 4.** Relation between *m* and IMD values with different number of stages.

Fig. 5. Relation between *m* and number of stages to achieve an IMD of 5%.

Fig. 6. Error power spectra between original input and output using RSSB-AM with different stages (a) $m = 0.5$ and (b) $m = 0.75$.

values of the modulation index and at large number of iterations of the RSSB-AM.

Figure 5 demonstrates the relation between the modulation index and the number of iterations (up to twelve iterations) to achieve an IMD of 5%. This figure shows that the iterative process of the RSSB-AM permits modulation index with larger values (which generates higher amplitude of the desired signals) at a given IMD value. Furthermore, we observed that possible values of *m* increases for the first five iterations, after which a reduction of values of *m* is observed. So using the modulation index of 0.75 at five iterations strikes a good balance between the amplitude of the desired signals and a low IMD value of 5%.

3.2. Error Analysis

In this subsection, the error analysis of the RSSB-AM using one speech signal is presented. Based on the
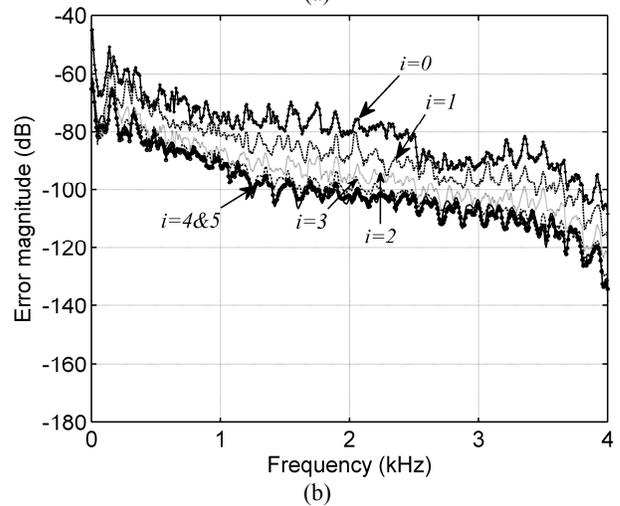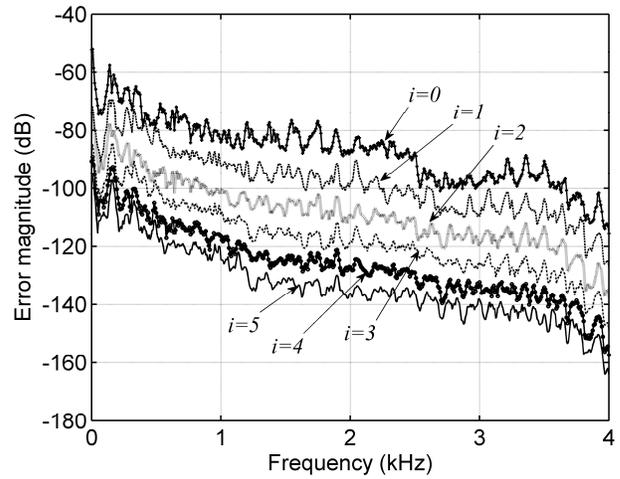
previous IMD analysis, we chose two modulation indices of 0.5 and 0.75 to avoid overmodulation. The demodulated signal $y_i(t)$ obtained with the RSSB-AM having different number of iterations ($i = 0, 1,..., 5$) is compared with the original input signal $x_0(t)$ to obtain the error signal which is $d_i(t) = y_i(t) - x_0(t)$, where $i = 0$ is the case of the SSB-AM. The error power spectrum $D_{m,i}^2(\omega)$ is computed using a 1024-point Hanning-windowed short-time FFT. A normalized speech signal of duration of 2.56 s is used as the test input. The average error power spectra of the RSSB-AM having different number of iterations are plotted in Fig. 6.

From Fig. 6, it is obvious that the RSSB-AM having more iterations suppresses more distortion, hence this technique is capable to reproduce a demodulated signal closer to the original input signal, especially when $m = 0.5$. According to Table 1, the distortion in the *i*th

iteration of the RSSB-AM can be divided into two groups: first group G_1 with the minimum order of the modulation index, whose value decreases with more iterations and second group G_2 with all the rest of the distortion terms whose value increases with more iterations for larger modulation index. It is obvious that terms in G_1 are dominant in the error signal for the first few iterations ($i \leq 2$). The ratio between these two modulation indices in the error spectrum $20\lg|D_{m2,i}(\omega)/D_{m1,i}(\omega)|$ for $i \leq 2$ is approximately $20\lg(m2/m1)^{i+2}$, where $m1$ and $m2$ are two modulation indices used in this subsection. This observation holds when the number of iterations is increased for smaller modulation index. In Fig. 6(a), the error magnitude is almost inverse proportional to the number of iterations. However, this is not the case when a larger modulation index is used as shown in Fig. 6(b). From Fig. 6(b), G_2 is found to increase and become equal or larger than G_1 after the first few iterations. This leads to a larger value of $20\lg|D_{m2,i}(\omega)/D_{m1,i}(\omega)|$. For example (referring to Fig. 6), considering $\omega = 2\pi(1k)$, the ratio of $20\lg|D_{m2,i}(\omega)/D_{m1,i}(\omega)|$ is 10.6 dB when $i = 1$, which is almost equal to $20\lg(m2/m1)^3$. On the other hand, the ratio of $20\lg|D_{m2,i}(\omega)/D_{m1,i}(\omega)|$ is 32.6 dB when $i = 5$, which is much larger than $20\lg(m2/m1)^7$. When the increment in error spectrum caused by G_2 is almost equal to the decrement from G_1, the error spectrum remains almost constant despite of more iterations used. As shown in Fig. 6(b), there is little difference in error spectrum using four and five iterations. For a much larger m (such as $m = 0.9$) and more iterations (such as $i = 6$), the increment of G_2 is significantly higher than the decrement from G_1, which leads to $D_{m,i+1}(\omega) > D_{m,i}(\omega)$ and hence the iterative process of the RSSB-AM should stop. As seen in this subsection, a moderate modulation index should be adopted in the RSSB-AM to reproduce an accurate original directional signal while minimizing the risk of severe overmodulation.

4. CONCLUSIONS

The motivation of the RSSB-AM algorithm and its performance analyses for parametric loudspeakers were presented in this paper. An illustrative explanation of the RSSB-AM demonstrates how distortion can be reduced iteratively and the effect of the number of iterations to distortion reduction performance has been discussed.

Numerical simulations for speech reproduction showed that the RSSB-AM having one iteration can outperform the conventional DSB-AM. Based on the

IMD and error analyses, the modulation index should not be greater than 0.75 for the RSSB-AM having five iterations. A computational complexity and performance tradeoff have also been investigated, so that the most efficient implementation of the RSSB-AM can be selected for different modulation indices. The observed features of the RSSB-AM algorithm provide a useful guideline in improving the performance of parametric loudspeakers.

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