Global Minimum Peak RF Design for Large Time-Bandwidth Saturation Pulse

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Introduction

Criteria for desirable saturation profile are flat passband and sharp profile with minimum peak \( f(t) \) amplitude. Design parameters for RF pulses include bandwidth \( \Delta \) and stopband \( \delta \) ripple tolerances and time-bandwidth product \( tb \). [1-4]

The well-known Shinnar-Le Roux (SLR) RF pulse design technique is a transform that relates magnetization profile to two polynomials \( A_{\Delta} \) and \( B_{\delta} \). \( B_{\delta} \) is traditionally obtained by digital filter design techniques.

A conventional approach (for minimum-peak \( f(t) \)) is to design a maximum-phase \( B_{\delta} \). However, \( B_{\delta} \) to obtain its roots, then combinatorially search for each root inversion \( G \) over all possible phase patterns. But this conventional method is limited to \( tb \approx 18 \) before number of combinations becomes prohibitive.

Our method uses convex optimization to design global minimum peak RF saturation pulse. The use of autocorrelation matrix \( B_{\delta} \) overcomes the non-convex passband ripple constraint. But the rank of this autocorrelation matrix has to be 1, which again is non-convex. Our solution is to use convex iteration.

Methods

For saturation pulses, the RF pulse \( f(t) \) and \( B_{\delta} \) are very much alike.

Our technique: use optimization to generate a minimum peak \( B_{\delta} \), then the saturation profile \( |M_{\delta}| \) and RF pulse \( f(t) \) are found via SLR transform as in the conventional method.

Our problem statement:

\[
\text{minimize} \quad \| B_{\delta} \|_2
\]

subject to \( 1 - \delta_1 \leq \|B_{\delta}\|_2 \leq 1 + \delta_1, \quad \omega \in [\omega_1, \omega_2] \)

\[
[\text{Eq. 1}]
\]

where \( B_{\delta} \) is the Fourier Transform of \( B_{\delta} \). But this problem statement is nonconvex (because of the left-hand side of the first constraint).

So instead, define an autocorrelation matrix of \( B_{\delta} \) as \( G = B_{\delta}B_{\delta}^H \) where \( G \) is positive semidefinite with rank 1.

Summing along each of \( 2N - 3 \) subdiagonals produces entries of the auto-

correlation function \( r(t) \) of \( B_{\delta} \) where (Fig. 2)

\[
r(t) = \sum_{j=0}^{N/2} r_{\text{max}} \frac{\cos(j \omega t)}{(1 + \delta_1)^2 \omega \leq \|B_{\delta}\|_2 \leq 1 + \delta_1)} \quad \omega \in [\omega_1, \omega_2]
\]

\[
[\text{Eq. 2}]
\]

Convex Iteration for rank constraint

The idea of convex iteration is to rewrite Eq. 2 as a sequence of convex optimization problems by introducing direction vector \( W \).

\[
\text{minimize} \quad \| \text{diag}(G) \|_\infty
\]

subject to \( (1 - \delta_1)^2 \leq \|B_{\delta}\|_2 \leq (1 + \delta_1)^2, \quad \omega \in [\omega_1, \omega_2] \)

\[
[\text{Eq. 3}]
\]

Discussion

We presented a method to design an RF saturation pulse having globally mini-

mum peak RF power without the need for exhaustive root-inversion search. To formulate the problem as convex, we employ the tricks of autocorrelation matrix and convex iteration.

![Graphical abstract](https://example.com/abstract.png)